



INSTITUTE OF MATHEMATICS EDUCATION

Junior Maths Olympiad 2025 (Higher Level)

Std.: VII and VIII

Question paper

Date: 02.02.2025

Time: 2 Hours

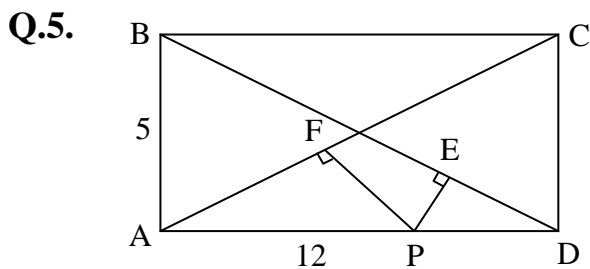
Total Marks: 100

Q.1. A rack has 6 different pairs of shoes. Find the number of ways of choosing 4 shoes from it so that there will be no complete pair. **(6 marks)**

Q.2. If ${}^{20}C_{10}$ is factorized as $(2)^p \cdot (3)^q \cdot (5)^r \dots$, Find $p + q + r$. **(6 marks)**

Q.3. If set $A = \{(x, y) | x^2 + y^2 \leq 50, x, y \in \mathbb{N}\}$ and set $B = \{(x, y) | 5x + 7y \leq 50, x, y \in \mathbb{N}\}$, then find (i) $n(A \cap B)$, (ii) $n(A \cup B)$, (iii) $n(A \Delta B)$ **(6 marks)**

Q.4. If $a, b, c > 0$, then prove that $\frac{a^3}{b} + \frac{b^3}{c} + \frac{c^3}{a} \geq ab + bc + ca$. **(6 marks)**



ABCD is a rectangle where $AD = 12$, $AB = 5$. The point 'P' is a point on AD such that PE is perpendicular to BD at E and PF is perpendicular to AC at F. Find $PE + PF$.

(6 marks)

Q.6. Consider set $X = \{1, 2, 3, \dots, 100\}$. Find number of ways when one can select 3 distinct numbers from set X so that product of the chosen numbers is divisible by 49. **(8 marks)**

Q.7. What is the maximum possible value of K for which 2025 can be written as a sum of K consecutive positive integers? **(8 marks)**

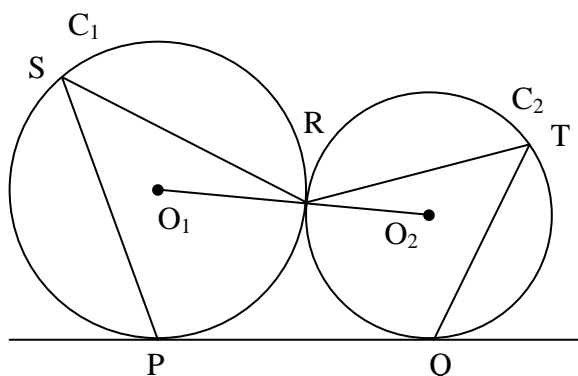
Q.8. (i) How many ordered pairs (p, q) where p and q are natural numbers can be formed such that $L.C.M(p, q) = 1800$ and $G.C.D(p, q) = 6$. (ii) Write all pairs. **(8 marks)**

Q.9. The quadratic equation $x^2 - px + q = 0$ has two real roots α and β . **(8 marks)**

(a) Find the quadratic equation with roots α^3, β^3 .

(b) If new quadratic equation [obtained in (a)] is still $x^2 - px + q = 0$, then find all possible pairs of (p, q) .

Q.10.



Refer figure. PQ is the common tangent to the circles C_1 and C_2 with centres O_1 and O_2 respectively. These circles touch externally at point R. Two chords of circle C_1 are PS and SR and two chords of circle C_2 are QT and RT. Find sum of measures of $\angle PSR$ and $\angle QTR$.

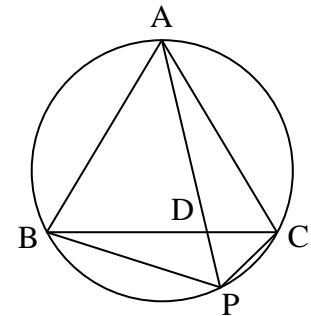
(8 marks)

Q.11. A pack contains n cards numbered 1 to n . Two consecutive numbered cards are removed from the pack. The sum of numbers on the remaining cards is 595. If the smallest of the numbers on the removed cards is k , find the value of k . **(10 marks)**

Q.12. If $f(x) + f\left(\frac{x-1}{x}\right) = x$, find (i) $f(x)$, (ii) $f(2)$ **(10 marks)**

Q.13. Refer Figure. A line drawn from the vertex A of an equilateral triangle ABC meets BC at D and the circumcircle at P . Prove that **(10 marks)**

(i) $PA = PB + PC$ (ii) $\frac{1}{PD} = \frac{1}{PB} + \frac{1}{PC}$



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