# INSTITUTE OF MATHEMATICS EDUCATION
## JUNIOR MATHS OLYMPIAD – 2024 (Primary Level)

Std. : VII and VIII **Question Paper** Date : 04.02.2024

Time : 2 Hours Total Marks : 100

**Q.1.** Three numbers $A$, $B$, $C$ are in geometric progression with common ratio $r$. Also numbers $A$, $2B$, $C$ are in arithmetic progression. Determine the sum of all possible values of $r$. **(6 marks)**

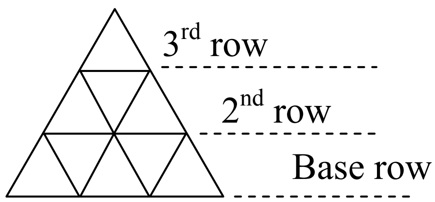
**Q.2.** The difference between the H.C.F and the L.C.M of x and 18 is 120. Find x. **(6 marks)**

**Q.3.** The equations have $ax^2 - bx + c = 0$ and $ax^2 - (b + 6)x + 3c = 0$ have one root common. Form the equation whose roots are the uncommon roots of the above given equations. **(6 marks)**

**Q.4.** A faulty odometer of a car proceeds from digit 8 to the digit 0 skipping the digit 9 regardless of the place value of the position where 8 occurs. For example travelling 1 km, the odometer changed from 000038 to 000040. If the odometer now reads 002010, find the actual distance travelled by the car. **(6 marks)**

**Q.5.** If a, b, c, d are four positive real numbers, then find minimum value of $\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} + 5$ **(6 marks)**

**Q.6.** A large equilateral triangle is constructed by using toothpicks to create rows of small equilateral triangles. For example, in the figure we have 3 rows of small congruent equilateral triangles, with 5 small triangles in the base row. Total N number of toothpicks would be needed to construct a large equilateral triangle if the base row of the triangle consists of 2003 small equilateral triangles. Then find the sum of all digits of N. **(8 marks)**




**Q.7.** Find the sum of all the positive integers less than 2009 and relatively prime to 2009. **(8 marks)**

**Q.8.** The number 2024 has the property that its unit digit is the sum of its other digits, i.e. $2 + 0 + 2 = 4$. How many integers less than 2024 but greater than 1000 share this property? **(8 marks)**

**Q.9.** If $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1 x + a_2 x^2 + \ldots + a_{50} x^{50}$, then

i) Find $\frac{a_2}{a_0}$ ii) If $a_4 = k \times 10^{50}$, find k. **(8 marks)**

**Q.10.** In a right angled $\Delta PQR$, hypotenuse is PR. A circle with side PQ as diameter is drawn to intersect the side PR at point L. Let tangent at point L to the circle intersect the side QR of the triangle at point M and length of LM is 5. Then value of

QM + MR + LM = **(8 marks)**

**Q.11.** Let ABCD be a square and let P be a point on CD such that DP : PC = 1 : 2. Let Q be a point on AP such that $\angle$ BQP = 90°. If $\frac{\text{area of quadrilateral PQBC}}{\text{area of square ABCD}} = \frac{m}{n}$ (where m, n are co-prime to each other), then the value of m + n is **(10 marks)**

**Q.12. (i)** Let $f(x) = \frac{x^{198}}{x^{198} + (1-x)^{198}}$, then find 'r' where

$$f\left(\frac{1}{199}\right) + f\left(\frac{2}{199}\right) + f\left(\frac{3}{199}\right) + .... + f\left(\frac{198}{199}\right) = r$$

**(ii)** If $\frac{1}{2\sqrt{1}+\sqrt{2}} + \frac{1}{3\sqrt{2}+2\sqrt{3}} + \frac{1}{4\sqrt{3}+3\sqrt{4}} + ... + \frac{1}{(r+1)\sqrt{r}+r\sqrt{r+1}} = \frac{p}{q}$

[Use value of r from Q. 12(i) and p, q are co-prime to each other], find (p + q)

**(10 marks)**

**Q.13** A natural number is called to be super star number if the number is less than 10 times the product of its digits. Find how many super star numbers are there from 10 to 200.

**(10 marks)**