INSTITUTE OF MATHEMATICS EDUCATION
JUNIOR MATHS OLYMPIAD – 2023 (Higher Primary Level)

Std. : VII and VIII **Question Paper** Date : 05.02.2023

Time : 2 Hours Total Marks : 100

Q.1. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. Find the number of common terms in these two series up to 10000. **(6 marks)**

Q.2. A square board of (6 × 6) cm is divided in 36 equal squares. In how many ways 2 squares can be selected such that they are not in the same row or column. **(6 marks)**

Q.3. If α, β and γ are the roots of $2x^3 + 5x^2 - 6x - 9 = 0$, then find $\alpha^3 + \beta^3 + \gamma^3$ **(6 marks)**

Q.4. Find the remainder when $1^6 + 2^6 + 3^6 + 4^6 + 5^6 + 6^6$ is divided by 7. **(6 marks)**

Q.5. Find the coefficient of x^7 in $(1 + x)^3 (1 + x^2)^9$. **(6 marks)**

Q.6. If a, b, c are positive real numbers, then prove that $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$. **(8 marks)**

Q.7. In a group of 10 persons, each person is asked to write the sum of ages of other 9 persons. If all the 10 sums form a 9-element set – {82, 83, 84, 85, 87, 89, 90, 91, 92}. Then find the age of the eldest person. **(8 marks)**

Q.8. In a quadrilateral ABCD, it is given that AB = AD = 13, BC = CD = 20, BD = 24. If r is the radius of circle inscribable in the quadrilateral, then find value of m, n where $r = \frac{m}{n}$ (m, n are co-prime number) **(8 marks)**

Q.9. (i) Solve for x which satisfy $|x^2 - 9| + |x^2 - 4| = 5$ **(8 marks)**

(ii) Find the sum of all the real roots of the equation $|x-3|^2 + |x-2| - 1 = 0$

Q.10. There are 200 students in a school. Out of these, 100 students play cricket, 50 students play hockey and 60 students play basketball. 30 students play both cricket and hockey, 35 students play both hockey and basketball, and 45 students play both basketball and cricket. Then **(8 marks)**

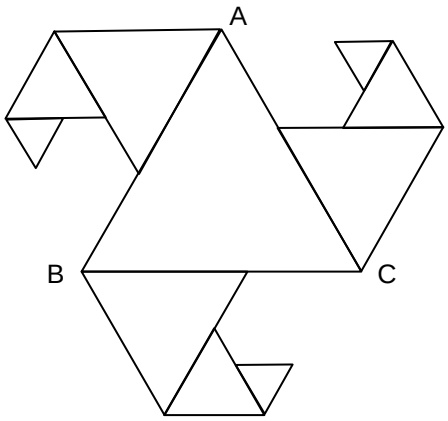
(i) What is the maximum number of students who play at least one game

(ii) What is the maximum number of students who play all the 3 games

(iii) What is the minimum number of students playing at least one game

(iv) What is the minimum number of students who play all the 3 games

Q.11. ABC is an equilateral triangle of side length 2^{100} units. Smaller equilateral triangles of side length 2^{99}, 2^{98}, 2^{97}, ……, $2^0 = 1$ are constructed on all the 3 sides. (as shown in the diagram for first 4 triangles).. If this figure is made by using a rope, then find the length of the rope required. **(10 marks)**



Q.12. A 391 digit number has the property that every two consecutive digits as a two digit number is a multiple of 17 or 23. Find the maximum sum of digits of such 391 digit number. **(10 marks)**

Q.13. As shown in the figure, ΔABC in divided into six smaller triangles by lines drawn from vertices through a common interior point P. The areas of these triangles are as indicated. Then **(10 marks)**

(i) value of m =
(ii) value of n =
(iii) Area of triangle ABC =