INSTITUTE OF MATHEMATICS EDUCATION

JUNIOR MATHS OLYMPIAD – 2022 (Higher Primary Level)

Std. : VII/VIII **Question Paper** Date : 12.02.2022

Time : 2 Hours Total Marks : 100

1. The coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 6 : 33 : 110, then n =

2. Find the sum of last three digits of $(97)^6$

3. If $x = \frac{\sqrt[3]{4}-1}{\sqrt[3]{2}}$, then $10x^3 + 30x + 2 =$

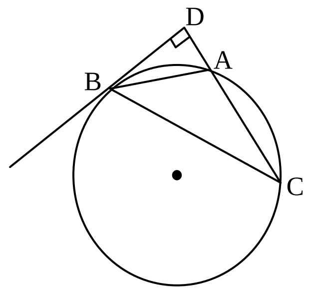
4. Let n be the smallest positive integer that is a multiple of 75 and has exactly 75 positive divisiors, including 1 and itself. Find $\frac{n}{1200}$

5. Find the remainder if 1! + 2! + 3! + … + 10! is divided by 11 and write the number you obtain when you add 11 to the remainder.

6. Find square of the remainder when 18^{220} is divided by 7.

7. What is the remainder when 2(26!) is divided by 29.

8. Let ABC be triangle such that AB = AC. Suppose the tangent to the circumcircle of ΔABC at B is perpendicular to AC extended at D, as shown in figure. Then value of $\angle ABC$ is (in terms of degree)
Note : Figure is not to scale.



9. Two sides of a triangle are $\sqrt{3}$ & $\sqrt{2}$ units. The medians to these two sides are mutually perpendicular. If length of third side is k, then the value of 21 k is

10. There are 100 students in a class. In an examination, 50 of them failed in Mathematics, 45 failed in Physics, 40 failed in Biology and 32 failed in exactly two of three subjects. Only one student passed in all subjects. Let the number of students failing in all three subjects be k, then value of 10 k is

11. Let f be a function defined from $R^+ \to R^+$ (Here R^+ denotes the set of positive real numbers.)
If $(f(xy))^2 = x(f(y))^2$ for all positive numbers x & y and f(2) = 6, then value of f(50) is

12. If x, y, z are positive real numbers, then square of minimum value of $\frac{(x+y)(y+z)(z+x)}{xyz}$, is

13. Let ABCD be a rectangle and Let E and F be points on CD and BC respectively such that Area of $\Delta ADE = 16$, Area of $\Delta CEF = 9$ and Area of $\Delta ABF = 25$. Then the area of ΔAEF is

14. Let $a, b, c, d \in R^+$, then minimum value of $(a + b + c + d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$ is

(Here R^+ denotes the set of positive real numbers.)

15. If $x + y = 7 + \sqrt{xy}$, $x^2 + y^2 = 133 - xy$ and $x^3 + y^3 = 10k + 3$, then find k.

16. Solve the following equation and find the value of x.

$\sqrt{x-1} + 6\sqrt{\sqrt{x-1}} = 16$

17. If α and β are the roots of quadratic equation. $x^2 - 8x + 2 = 0$ and $\alpha^4 - \beta^4 = 15\sqrt{14}\ k$, then find the value of k.

18. Find the number of ways of selecting 3 distinct numbers from a set $\{3^1, 3^2, 3^3, 3^4, \ldots, 3^{101}\}$ such that the three selected numbers are in geometric progression. Let this number be N. Find $\frac{N}{100}$.

19. Let 'n' be a 5 digit number. When 'n' is divided by 100, we obtain quotient 'q' and remainder 'r'. If number of values of 'n' for which the sum of q and r is divisible by 11 is x, then find the sum of the digits of x.

20. Two professors and four students are to sit in a row for a photograph. Each professor must be accompanied by students on his both sides. Let the number of possible arrangements be k. Write $\frac{k}{2}$.

Answer key

1. [12]	**2.** [20]	**3.** [17]	**4.** [27]	**5.** [11]
6. [16]	**7.** [28]	**8.** [30]	**9.** [21]	**10.** [20]
11. [30]	**12.** [64]	**13.** [30]	**14.** [16]	**15.** [79]
16. [17]	**17.** [64]	**18.** [25]	**19.** [18]	**20.** [72]

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