



INSTITUTE OF MATHEMATICS EDUCATION

JMO 2020 (Junior Mathematics Olympiad) – Std. VII and VIII
Maximum Marks : 100

Date : 02/02/2020

Time : 10.00 am to 12.00 pm

1. Show that 2^3 always divides the product of sum and difference of two odd natural numbers. (6 marks)
2. If $(x + 2)(x + 4)(x + 6)(x + 8) = 945$ and x is an integer, find value of x . (6 marks)
3. Find domain and range of the function $f(x) = \frac{2x^2 - x}{2x - 3x^2}$. (6 marks)
4. In a triangle ABC, $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ac}$, then prove that $\angle A = 90^\circ$ (6 marks)
5. Find the sum of even divisors of 2020. (6 marks)
6. If the coefficients of the second, third and fourth terms in the expansion of $(1 + x)^{2n}$ are in arithmetic progression, then find the value of n . (8 marks)
7. An examination consists of 160 questions. One mark is given for every correct answer. If one fourth mark is deducted for every wrong answer and half mark is deducted for every question left unsolved, the student scores 79 marks. However, if half mark is deducted for every wrong answer and one fourth mark is deducted for every question unsolved, then the student scores 76 marks. Find the number of questions student attempted correctly, solved wrong and left unsolved. (8 marks)
8. LCM of a and 36 is 900. Write all possible values of a . GCD of b and 36 is 9. Write atleast five values of b . Here $a, b \in \mathbb{N}$. 'A' is the biggest possible value of 'a' less than 900 and 'B' is the smallest possible value of 'b' greater than 9. Find LCM and GCD of A and B. (8 marks)
9. Let $\triangle ABC$ be an acute angled triangle and CD be the altitude through C. If $AB = 8$ units and $CD = 6$ units, find the distance between the mid – point of BC and mid point of AD. (8 marks)

- 10.** Study the following number series and find the next two numbers in each series. (8 marks)
- (A) $H_{17} = 476, 629, 782, 935, 1088, \dots$
 (B) $H_{21} = 399, 588, 777, 966, 2289, \dots$
- 11.** (i) In how many ways, letters of the word 'SANTANU' can be arranged? (10 marks)
 (ii) In how many arrangements, two A's are not together?
 (iii) In how many arrangements, neither two A's nor two N's are together?
- 12.** Three circles C_1, C_2, C_3 with radii r_1, r_2, r_3 ($r_1 < r_2 < r_3$) respectively are given. They are placed such that C_2 lies to the right of C_1 and touches it externally. C_3 lies to the right of C_2 and touches it externally. Further there exists two straight lines each of which is a direct common tangent simultaneously to all three circles. (10 marks)
- (i) Draw figure
 (ii) Find values of $\frac{r_2 - r_1}{r_2 + r_1}$ in terms of r_2 and r_3
 (iii) Prove that $r_2^2 = r_1 r_3$
- 13.** Find the remainder if ${}^{20}C_{10}$ is divided by 169. (10 marks)

Answers

- 1.** Hint : Let the numbers be $2a + 1$ and $2b + 1$
2. $1, -11$
3. Domain = $\mathbb{R} - \left\{0, \frac{2}{3}\right\}$
 Range = $\mathbb{R} - \left\{\frac{-3}{2}\right\}$
4. Hint : Use cosine rule.
5. 3672
6. 3.5
7. Correct questions = 100
 Wrong questions = 36
 Unsolved questions = 24
8. LCM = 1350, GCD = 9
9. 5 units
10. (A) 1394, 1547 (B) 2478, 2667
11. (i) 1260 (ii) 900 (iii) 660
12. (ii) $\frac{r_3 - r_2}{r_3 + r_2}$
13. 3

